Interpretations of Fractions

We've seen in previous topics how the way that we write numbers in our Hindu-arabic numeration system with place value and zeros is not necessarily intuitive. Now imagine you're a child and just getting used to this system, and that you're then told that to represent numbers smaller than 1 or between those counting numbers, you need to write numbers called "fractions" and that they look like this.

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And you have to convince yourself that when you look at this arrangement of numbers (and the vinculum in-between), it doesn't refer to a number that's very much like 2, or like 3, but rather a number that's less than a whole unit.

After some time building the intuition that increasing numbers refers to increasing amounts, you now have to get used to the idea that increasing the bottom number, that's the denominator, actually makes the number smaller. On the other hand, increasing the numerator, that's the top number, doesn't have this effect.

Actually, fractions can be used to refer to different kinds of scenarios, which over time, probably adds to the confusion.

When we start to learn about fractions, we usually learn about them as representing parts of a whole. In this interpretation, the denominator indicates the size of the pieces, which corresponds with how many equal parts a whole would be cut into, and the numerator represents how many are there in the quantity. So you can have 1 half, 1 third, but then also 2 thirds, 3 quarters, 3 eighths. But this isn't all that fractions are, and the fact that we often use pies or pizzas to represent them means that we sometimes forget that fractions are also a way of representing numbers – and so also refer to values on the number line.

If the numerator is larger than the denominator, we call this an "improper fraction" and proper fractions are those where the numerator is smaller. In this case, say, if we have 11 eighths of a pizza, that might sound like it doesn't make sense. How can we have 11 if there are only eighths? But remember that the 8 just refers to the size of the pieces.

When we learn to add with fractions, the parts-of-a-whole view can help us to understand why we can't add say, 2/3 and 1/2 as they are. We need to first find a common way of expressing both of the numbers, so that it's meaningful to add them together.

However, there are other ways of thinking about fractions that become more common as we study maths later on. Suppose I say that I want to share 3 loaves of sourdough amongst my 4 friends. I don't have enough to give them one loaf each. Now if I had 8 loaves, it would be simple, I'd be able to work out 8 divided by 4 to give 2 – and that's how many each would receive. Unfortunately, I only have 3 loaves, but I can use the same approach: 3 divided by 4. As it turns out, this will give me exactly ¾ of a loaf each and it's no coincidence. In the division interpretation of fractions, 3 quarters means that we start with

3 whole things, and divide by 4. It refers to the same quantity, but it's a different interpretation to thinking of 1 whole, cutting it into quarters, and taking 3.

The division approach to interpreting fractions becomes more and more important later on when students start learning about algebra and we do away with division symbols.

The other way that we can think about fractions is as a ratio. If I make a hot chocolate in the ratio of 3 parts chocolate to 4 parts milk, I can say that there's 3/4 as much chocolate as there is milk, or that there is 4/3 as much milk as there is chocolate. I can also talk about the ratio of each part to the whole, so 3/7 chocolate and 4/7 milk.

Sometimes the differences between these interpretations are subtle and we need to always be mindful of what the "whole" is.

Being able to solve problems in mathematics often requires us being comfortable moving between different representations and being able to use the interpretation that best serves our intuition.

Fractions are just one tool for doing this, and they can be confusing, but their usefulness in being able to represent arbitrary quantities and amounts, and the methods we have at hand to add, subtract, multiply and divide them, probably give a good part of the reasoning as to why we didn't do away with them when we started using decimals.