[Tree diagrams and the multiplication rule]

Recall from our study of graphs that a tree was a graph with no cycles, however we can use special trees to describe paths of sequential events and determine the sample space.

For example, if we toss three coins, we can represent the paths like this.

Following each path along the tree leads to the set of outcomes and from this, we can determine the probability of different events. For example, there is a 3/8 chance of obtaining two heads and a tail, which is the same probability as tossing two tails and a head.

If we were rolling three dice, a tree might start to get a little big, however we can still use trees to model certain sets of outcomes by allocating different probabilities to the branches. For example, suppose we wanted to determine the probability of rolling at least 1 6 from three rolls. In this case, we are only interested in two outcomes for each roll, either a 6 is rolled (with a 1/6 probability) or a 6 is not rolled (with a 5/6 probability).

We can use almost the same diagram to determine the different sequences of rolls. We'll use a 6 to denote a roll of 6 and N to indicate that a 6 was not rolled.

We can see here that a 6 is only not rolled in one of the final outcomes, however we need to remember that these outcomes don't occur with equal probability like they do with the coin tosses. We can indicate the probability of each step by writing the probabilities next to the edges.

Now we are only left to determine the total probability of this bottom path in the tree – because if we can work out the probability of rolling NO sixes, then whatever is left over must be the probability of rolling at least 1 six.

To determine this, we use the multiplication rule, multiplying along the lines of the tree, 5/6 times 5/6 times 5/6. The reason why we use multiplication can be thought of like fractions. We're saying that out of the 5/6 rolls where we don't expect to roll a 6, 5/6 won't roll a six on the second rule, so rolling no sixes in two rolls is 5/6 of 5/6. Then out of this proportion where no sixes were rolled on the first two rules, we expect that 5/6 of these won't roll a six on the last roll.

So the probability of rolling no sixes is 125/216 and this means the probability of rolling at least 1 six is 91/216, because 91 + 125 = 216.

So it's more likely, rolling 3 dice, that you won't roll ANY sixes, than it is that you'll roll 1, 2 or 3.