

[Spanning Trees]

Suppose we have a graph representing a set of cities and we want to look at how to connect the cities to power or fast internet in the most cost-effective way. In this situation, an edge now represents the possibility of connecting any two cities directly, but we also introduce a weight to this edge, in our case representing the cost of using that edge. Now, the weight or cost of these edges could be based on the distance between cities, but it certainly doesn't have to be – and so, as before, the actual positions or lengths of the arcs are meaningless.

In graph theory, a tree is a graph that has no cycles. We further have the idea of a “spanning tree”, which is both a tree and a sub-graph of a particular network that keeps all of the vertices connected. In the case of connecting cities, we would be hoping to minimise the cost, and so we are looking for a minimal spanning tree, the tree that connects the vertices with the smallest weight in total.