[Minimum spanning tree example]

For the problem of finding a minimum spanning tree, our aim is to find a subset of edges that minimises the cost, while keeping all the vertices connected. Recall that a tree is a graph with no cycles, and so will be comprised of the minimum number of edges required to ensure that there's a path between any pair of vertices.

These two algorithms allow us to approach the problem systematically.

In Algorithm 1, we go through the edges one by one, and discard any edges that form cycles. This way, we're looking to build our tree from the cheapest edges possible.

We can list the edges in order to help us keep track.

Once we consider this edge, we can see that it forms a loop, and so it won't be required in the final tree. We can hence discard it and continue on. Some of the loops can end up being comprised of 4 or more edges. For small problems, we'll hopefully be able to spot these ourselves, but you can imagine that for much larger problems – like 1000 vertices, identifying the loops might become difficult, even for an algorithm implemented by a computer.

Once we've considered the final edge, we have our minimum spanning tree. This is not necessarily unique (we could swap the 13 edges in the middle) but it should achieve the minimum cost possible. In this case, the cost is 128.

We'll now implement an alternative algorithm on the same graph.

In this case, we proceed from most expensive to least expensive, and discard unless it causes any of the vertices to become disconnected.

Once we consider the 18 edge, we can see that removing it would separate the top half of the graph from the bottom, so this is the first edge that we need to keep.

At the end, we should achieve the same minimum cost.

There are other approaches to finding the minimum spanning tree too, and sometimes it'll depend on the graph you're considering, for instance, depending on whether there are more edges to keep or eliminate. In this case, we had 11 vertices and so the tree required 10 edges, and since we started with about 20, both approaches were similar.