[Euler path and cycle examples]

Here we'll give some examples for determining whether or not a graph contains an Euler path or cycle, and how to find them.

The problem is determining whether we can trace a path that crosses each edge exactly once. And recall Euler's result, that such a path can be found as long as the graph has a maximum of 2 odd vertices.

If there are no odd vertices, then we can find an Euler cycle and if there are 2 odd vertices, the Euler path will start at one and finish at the other.

So coming back to this first graph, we can first label each of the vertices with their degree. Then we can identify that in this case, we have two vertices which have an odd degree. Now starting from either of these points, it's relatively simple to find a path that works.

But now we need to describe and communicate our solution, and so for this, we introduce notation to keep track of the paths. We could do this by labelling the edges, however it's customary to label the vertices and then refer to edges by the pairs of vertices they run between.

And so now if we start our path at H, we can go HHFE CABC DG back to E, HIGJI

Now we can count the string of letters we have, 16 and note that this corresponds with 15 edges, so our path's the right length, and we could also just check that each path is represented by a pair of letters.

This isn't the only Euler path here, we could have started at I and gone in the other direction, and there are a number of other possible combinations, but all of them need to run between H and I.

How about this graph? Is there a route we can take that only travels along each road once?

It appears to be much simpler than the previous graph, but when we enumerate the degree at each vertex… we can see that there's too many odd vertices.

How about these two graphs? Will there be an Euler path in either of them?
In the first graph, there are 4 odd vertices, so again, we won't have an Euler path and on the right, we only have two – so in this case, we'll be able to find an Euler path, starting from one and ending at the other.

How about this graph? Can you find an Euler path?

It's a bit more tricky than the Yes and No graphs, so let's label write out the degree at each vertex.

So now all of the vertices are of even degree. That means that there will be an Euler Cycle – which is an Euler path that starts and finishes at the same vertex. And in fact, we can choose any vertex to start at, and as long as we are careful not to get ourselves stuck, we should be able to find a path that ends where we started.