[Volume of a pyramid]

How does the volume of a pyramid relate to its height and the area at its base? Let's do some investigating. We can consider some of the old stepping pyramids, which should be much easier. The idea is that each step of the pyramid will be a square with uniformly decreasing side length.

Let's suppose that we have a pyramid that's 10 units across the base, the question is, what will be the relative step sizes if we go upwards 5 times to reach the apex?

Well looking from the side, we can see that these are actually similar triangles, which means that the length of the base will reduce proportionally to the decrease in height. If the Pyramid started off at 5 units high, the ratio will be 5:10 or 1:2 for each step. So when the height here is 4, the step size will be 8, the next step will be 6 and so on.

This gives us a volume of 10^2 + 8^2 + 6^2 + 4^2 + 2^2 = 100 + 64 + 36 + 16 + 4 = 220, and that compares to a cube, which with the same base would be 10 x 10 x 5 = 500 units in volume.

Let's increase the number of steps.

Now we'll have 10^2 + 9^2 + 8^2 + 7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 divided by 2, since each of these slabs is only half a unit high. And in this case, we get 192.5 and we could keep on going and going,

And now if we continue in this fashion, decreasing the step sizes so that we have more and more slabs and they'll look more and more like a real pyramid, the ratio of the volume of these pyramids to the volume of a corresponding polyhedron with the same base and same height, will get closer and closer to a third.

So whether it's a pyramid, a cone, or some other similar shape, as long as it's cross sectional area is decreasing at the same rate, then its volume will be 1/3 x base x height. This formula is actually used by arborists to estimate the volume of branches of trees.