[Triangle algebra]

Let's place our triangle with lengths a,b, and c on the ground so that c is our base. To calculate the area, we just need to work out the height it reaches, so we can draw in this line and we'll call it z. So z is perpendicular to c.

Now let's introduce two new values, x and y which represent the two parts of c that fall either side of the line. From Pythagoras's theorem, we have:

$$x^2 + z^2 = a^2$$
$$y^2 + z^2 = b^2$$

and we also know that $x + y = c$.

Now the tricky thing here is to rearrange these bits of information so that we can work out z, just from knowing a,b,c.

Now of course, if we can find x or y, we can also find z, but the key is that we need to be able to find any one of them, only knowing a,b and c.

As it turns out, we can do this, and one way is via the value of x.

First, we know that using pythagoras's theorem, the height as determined from a and x needs to be the same as the height determined from b and y. So that gives us:

$$b^2 - y^2 = a^2 - x^2$$

and we can rearrange this so that we have the parts we'll know on the right (remember that we're assuming we already know a, b and c).

$$x^2 - y^2 = a^2 - b^2$$

Now this is starting to look promising, because since we know x and y add together to make c, we can actually express y in terms of c (which we'll know and x)

So x + y = c, y = c – x, and from this:

$$a^2 - b^2 = x^2 - (c - x)^2$$

We can expand these brackets:

$$a^2 - b^2 = x^2 - (c^2 - 2cx + x^2)$$
$$a^2 - b^2 = x^2 - c^2 + 2cx - x^2$$

and this is looking better and better because now we can eliminate x^2. And we're just left with

$$a^2 - b^2 = -c^2 + 2cx$$

which we'll rearrange

$$a^2 - b^2 + c^2 = 2cx$$

$$\frac{a^2 - b^2 + c^2}{2c} = x$$

So this means that if we know a, b, and c, we'll be able to find x straight away, and from there it's only one more step to find the value of z and hence be able to calculate the area.