[Base conversion]

A place-value system means that the position of a symbol can change it's value. So the 2 in 14023 has a different value to the 2 in 2004319. When using 10 symbols, we cycle through our symbols in order until we get to 9 and then we have "run out" of symbols to use in the units column. The crucial step we take at this stage is to use a 1 to represent 10 units (by placing it in the tens column) and then we can recycle the symbols in the units column. After we have 10+9 = 19, we can increase the 1 to 2 to represent two tens and so on.

Now consider what would happen if we have 5 symbols (0,1,2,3,4). In this case, it's once we get to 4 that we have run out of symbols and now we put a 1 in the next column. But this column doesn't represent 10 any more, it's now equal to 5 times as much as each unit. Once we have 1(x5) + 4, which is the same as 9 in our decimal system, we then place a 2 in that column, which now represents two fives. How about once we are up to 44 in base 5? This means 4 lots of 5 and 4 lots of 1, which is 24, and so now when we go to 100, the 1 represents 25 units (5 x 5). The pattern repeats the same for each column.

We can do this in other bases too – let's look at binary, which means base 2. The columns are worth 1, 2, 4, 8, 16, .. Each increase in one column means 2 times bigger, whereas with decimal every column was 10 times bigger.

An interesting thing you might notice, as might be the case with some imperical units, is that we lose intuition about how big a number is. How big is the base 5 number 4313 ? How big is the binary number 100011 ?

Let's have a look by converting.

4313 in base 5 means

$4 \times 5^3 + 3 \times 5^2 + 1 \times 5^1 + 3$
$= 4 \times 125 + 3 \times 25 + 5 + 3 = 500 + 75 + 5 + 3.$

100011 means
$1 \times 2^5 + 2 \times 2^1 + 1 = 32 + 2 + 1 = 35.$