[Index laws]

Index laws can be seen as related to multiplication in a similar way that multiplication is related to addition. That is, when we say 3 multiplied by 4, we can think of this as 4 additions of 3: 3+3+3+3 (or equivalently 3 additions of 4: 4+4+4). So algebraically, when we see the term 4a, we know that this means a + a + a + a and even if we have fractional values, like 5/3 a, we can extend our understanding that this is the same as a + 2/3 of a – if a is a length or a weight then it makes sense.

With indices when we say 3 to the to the power of 4, or 3 to the 4, this means 4 multiplications of 3: 3 x 3 x 3 x 3. So a to the power of 4 is a x a x a x a. Extending this to fractions and decimals is a little harder to picture, but for the moment we can note that 3^2.6 must be some value between 3^2 or 3 x 3 and 3^3 or 3x3x3. The notation we use was actually introduced by DesCartes (also a philosopher) and it's important to keep in mind that the powers operation isn't symmetric like multiplication, 3 multiplications of 4 is different to 4 multiplications of 3. We usually refer to the number above as the index or the power, while the number that we're taking multiples of is called the base.

So what about when we perform operations on terms involving indices? We want to be able to simplify terms.
Let's start with 3 to the power of 2 or 3 squared, which means 2 multiplications of 3.
3 x 3

if we increase the index, we'll end up with 3 multiplications of 3

3x3x3

and each increase of the power introduces an extra multiplication.

3 x 3 x 3 x 3 x 3
If we decrease the power, then we'll reduce the number of multiplications, so from 3 x 3 x 3 we get to 3^2 and then 3 by itself is just 3 ^ 1. This is an important identity law - similar to how 1a = a, we have that a^1 = a as well. So 10^1 = 10, 5^1 = 5 and 7^1 = 7.

Another way of thinking of this reducing the power, is that it's the same as dividing by the base. So this means we can go even further, from 3^4 we'll have 3^3, 3^2 = 3 x 3, 3 ^1 = 3, then 3^0 = 3^1 divided by 3 = 0, then let's keep going, 3^-1 = 1/3, 3^-2 = 1/9, 3^ -3 = 1/27 and so on. How about if we had 5s?
5^3 = 5x5x5, 5^2 = 5x5, 5^1 = 5, 5^0 = 5/5 = 1, 5^-1 = 1/5, 5^-2 = 1/25. So a couple of important things we notice here is that anything to the power of 0 is actually 1 (and not 0 or the number itself – even though in our original conception of indices, we thought of it as repeated multiplication and it might seem a little confusing to think of "no multiplications" as being equal to 1 – however remember that 1 plays an important role in multiplication, the same way 0 does with addition since 0 + a = a and 1 x a = a – so one way to think of it is that if 0 additions of a number gives you zero, then no multiplications of a number gives you 1.)
The other thing we see is the meaning of a negative index, 3^-2 means 1/3^2, 5^-2 = 1/5^2 and so on. The pattern is the same that we observe in the place value system with 10 as the base.

The pattern can also help us understand how multiplication and division work with indices. If you have the same base, e.g. 3^4 x 3^2, then we can write this out as 3 x 3 x 3 x 3 x 3 x 3, each power gives you an extra multiplication. So 3^m x 3^n = 3^(a+b). This still works if we have negative values, if we have 3^4 x 3^-2, then this is 3 x 3 x 3 x 3 x 1/3^2 or 3 x 3 x 3 x 3 x 1/3 x 1/3. Each positive amount gives you a multiplication, each negative amount gives you a division, so we'll have 3^4 x 3^-2 = 3^(4 + -2) = 3^2. However this doesn't work if we have different bases. If we have 3^4 x 2^2, then there's no simplification we can do. It also doesn't work if we're adding terms with indices. 3^4 + 3^2 doesn't simplify in terms of the index other than by factoring out – so you could write 3^2(3^2 + 1) but this might not really help you that much. So if we have the same base, then the rule is usually expressed as a^m x a^n = a^(m+n). And don't forget in the latter case that we're following order of operations where "order" comes before multiplication, division, addition and subtraction. It's as if we have brackets there.

For division of indices, let's consider 3^5 divided by 3^2. Written out, we have 3 x 3 x 3 x 3 x 3 div 3 x 3. It's important to remember that we're dividing the whole term, so this is different to 3 x 3 x 3 x 3 x 3 div 3 x 3. We're actually dividing by 3 for every power in the divisor. We might think of this in a similar way to cancelling out fractions, or alternatively we could express the whole equation as a multiplication. So we have 3^5 div 3^2 = 3^(5-2) = 3^ 3 and the general rule we have is a^m div a^n = a^(m-n). Again, this works with negative numbers too. If we have 3 ^ 5 div 3^-2, then we're dividing by 1/3^2 which is the same as multiplying by 3^2 and gives us 3 x 3 x 3 x 3 x 3 x 3 x 3, or 3^5 div 3^-2 = 3^(5 - -2) = 3^(5+2) = 7.

So summarizing the important rules we have...

a^1 = a
a^0 = 1
a^-m = 1/a^m or a^m = 1/a^-m
a^m x a^n = a^(m+n)
a^m div a^n = a^(m-n)

and remember with those last two that you have to have the same base. A good thing to do is, whenever you use the rule, to double check that it works with some easy numbers. So what's 10^2 x 10^3? Well with tens, it's just the number of 0s after the 1.
so that would be 10 x 10 = 100, and 10 x 10 x 10 = 1000 so together it's 100 x 1000 = 100,000 or 10^5