[Solving equations]

In the case of Mae's problem, we can write expressions for the cost using each of the methods. If Mae sends away, it will cost 2.50 per badge, so we will be multiplying 2.50 by the number of badges ordered. Since we're setting this up algebraically, we'll write it without the multiplication symbol as 2.50 b. We then add the 7 dollars for postage, and this expression will give the total cost.

For the hiring method, it only costs 20cents per badge, and we should express this in the same terms as previously, so we write it as 0.2 dollars per badge and add the 300 for the hiring cost.

Now one question we might ask is whether we should be using different variables for the badges in each case, or is it okay to use the same b? Well since we're comparing the price of badges, we can keep b the same, so if b =1, we'd have a cost from the first expression and a different cost from the second one, but both with respect to the cost of 1 badge.

However we might want to keep the cost variables separate, so we'll say P is the cost of using post and H is the cost of hiring.

Now for 1 badge clearly it will be better to use postage, but if we get 1000 badges it'll probably be better to use the machine hire since it's so much less per badge – so one thing we might do in the entry stage of this problem is to reframe the question as one that asks when the two methods will result in the same cost.

How can we work this out, well that's going to be when P is equal to H, so we set this up, and we're left with an equation that involves just numbers and b. Our task here is to solve for b. The way that we do this is we apply operations that will cancel out operations undertaken here until we're left with b equals something.

How do we choose these? We apply the inverse operations – the operations that are essentially the opposite of what is being undertaken. So if we subtract 7, we'll be able to cancel out on the left hand side because subtracting 7 is the inverse operation to adding 7, but then we need to make sure we apply that operation to both sides. Applying to both sides will mean that the equality in the middle will still hold.

Now we have b on both sides still, but we can get it on one side by subtracting all the b from the right hand side. Subtracting 0.2b from the right and from the left will keep the equality, and lead to a cancelling out on the right hand side.

Then finally we have 2.3 multiplied by b equals 293. To get b by itself, we do the opposite of multiplying by 2.3, which is dividing by 2.3 and then 2.3 over 2.3 will give b (remember that commutativity and associativity allow this to work even though the 2.3 is in front of the b and we're kind of applying the operation after the b).

And so now we have our final solution. When we're setting out our working for this kind of problem, it's good to annotate, making note of the operations we're applying each time and then rewriting the equation that we're left with. Sometimes we might like to further annotate by crossing out in places where we get a cancelling, or sometimes we prefer to leave these out altogether. That's fine, as long as it's clear and logical how we get to one line from the next.

In the case of Marisa's problem, we see straight away that having the two expressions is very helpful for this first motivation. For any number of badges, we can work out the total cost by simply substituting the number of badges for *b* in the equation. Recall that this is an example of specialising. So for 200 badges we will have,

$$C_H = 300 + 0.2 \times 200$$

$$= 300 + 40$$

$$= 340 \ ,$$

$$C_S = 7 + 2.5 \times 200$$
$$= 7 + 500$$
$$= 507 \ .$$

We see that hiring a badge-machine is cheaper in this instance. We could also use these equations to find out how much one can get for a given amount of money. For example, suppose we want to know how many badges we can get for $400 using either scheme. In this case, we can substitute the C_H or C_S for 400, but then we need to do a little rearranging to find out what *b* well be.

$$400 = 300 + 0.2b$$

Now looking at the right hand side, we can think of the order of operations and what happens to *b*. Firstly, *b* is multiplied by 0.2 (because multiplication comes before addition). Secondly, this value is added to 300. If we are to find out what *b* is then, we need to first work out what the value of 400 would be *before* we add the 300. We hence apply the inverse operation, subtracting 300, and we do this to both sides so that the equation symbol is still valid. At the moment there is 400 on the left-hand side, and the equals sign means that the expression on the right-hand side is also 400. Once we subtract 300, we will have

$$400 - 300 = 300 - 300 + 0.2b$$
$$100 = 0.2b$$

Remember that addition is associative and commutative, which is why we're able to subtract 300 (or add -300) from the 300 in the original equation without the 0.2*b* getting in the way.

We now have an equation that says 0.2*b* is equal to 100. Since b is being multiplied by 0.2, in order to work out what its value is, we can perform the inverse operation, dividing by 0.2 to both sides. This will give us,

$$\frac{100}{0.2} = \frac{0.2b}{0.2}$$

Equivalently, since 0.2 is the same as $^1/_5$, we could also have multiplied both sides by 5 to achieve the same result. We can evaluate the left-hand side to give 500. On the right hand side, what we actually have is $\mathbf{.2} \times \boldsymbol{b} \times \frac{\mathbf{1}}{\mathbf{0.2}}$. This can be arranged using the commutative and associative laws so that we have $\boldsymbol{b} \times \mathbf{0.2} \times \frac{\mathbf{1}}{\mathbf{0.2}}$ and since multiplying by 0.2 and by its reciprocal are inverse operations, they will give a result of 1 (which is why we chose to divide by 0.2 in the first place). So we will then have,

$$\mathbf{500} = \boldsymbol{b}\ .$$

If we then substitute *b*=500 back into our original equation, we should find that we get a result of 400.