[Introduction to algebra]

Algebra is sometimes seen as one of the threshold concepts of mathematics – you know, one of those concepts that people talk about as a point of departure,

"I was really great at maths and then **algebra** came along and I'm like, no way."

Other threshold concepts are things like fractions, decimals, calculus and derivatives and complex numbers – and it's worth noting that these roughly correspond with concepts that all of humankind struggled with – they're concepts that no-one had any thoughts about for a long time but then all of a sudden they became necessary or useful for explaining the world.

So it's fair enough that a lot of people check-out when it comes to algebra – because it requires an update to our mathematical thinking that previously was just getting comfortable working with numbers. Now we have to work with complete abstract quantities.

But like any paradigm shift, the use of algebra gives as a new insight into previous patterns that we might have been intuitively but not explicitly familiar with.

For example, did you ever notice that if you take a square number, for example $7^2=49$, well if you multiply the numbers either side of 7, 6 and 8, you get 48. Do it with any square. $10\times10=100$, and $9 \times 11 = 99$.

Well working with algebra makes this relationship quite unremarkable and completely expected.

$(x+1)(x-1) = x^2 + x - x - 1^2 = x^2 - 1^2$.

Working algebraically gives us access to other perspectives on a particular relationship too. For example, if you studied high school physics, you might have learned algebraic relationships to do with velocity, distance and acceleration. In particular, you can work out the square of the final velocity something is travelling at by taking the square of the initial velocity and then adding two times the acceleration times the distance. If we know all of these values on the right, we can just replace the symbolic letters with their values and then make the calculations, taking the square root at the end to undo the square. This kind of use of equations is called substitution.

So suppose a car is travelling at 2 m/s and it's accelerating over 10m. If it's accelerating at 1 m/s then the final velocity will be the square root of 2^2 + 2x1x10 or the square root of 24, a little less than 5.

But now suppose we want to know how fast we need to accelerate to get up to exactly 4. We could just do a little trial and error, but if this is a problem we'll need to work out again and again and again, it's much easier to rearrange this equation and express acceleration in terms of these other values.

One of the most useful skills in algebra is being comfortable with this sort of rearranging of equations, but actually what it relies on the most is a solid understanding of the rules of arithmetic. When people struggle with algebra – it's quite possible that it could be due to one of two things: as mentioned earlier, it requires a conceptual leap to start working with abstract symbols rather than numbers, then another problem is if someone has not obtained an adequate understanding of arithmetic and operations from previous study. For example, understanding that ab and ba are the same thing requires them to understand that 5x7 and 7x5 are the same thing. Being comfortable with 3a + 5a being equal to 8a also is easier if people have a good idea of general decomposition, and of course if people are weak on fractions, then being able to rearrange equations using division and multiplication gets much more cumbersome.

As we introduce algebra, part of our focus will be on how it will be useful for us later on, while part of it will be on making some of these rules and relationships explicit.