These two processes are a part of the transition between entry and attack, and between attack and review.

We've seen that in the entry stage, a great way to start a problem is to specialize. This means to look at a potential solution, a potential aspect of the problem, or a simpler version of the problem. As soon as we specialize, we start to understand the problem better – we begin to understand the mechanics of the problem, the underlying mathematics, what we might need to find out, and the potential difficulty of the problem.

We can randomly specialize – for example, in the Huashan problem, we just started moving pieces to understand the game. In the shopkeeper problem, specializing would include starting with a particular value and following the calculations through to see how much we end up with. While in the passing trains problem, we could look at some calculations after one hour and see whether that helps us gain some insight into the solution and with the married couples problem, specializing would be start with a particular arrangement to see how close it is to the 5000.

Random specialization is good in the beginning, but then for some problems, being random may never help us reach the solution and so we need to be a little more systematic. This will either help us identify a pattern, as with some of the examples in the handshake problem, or it will be a way of covering all possibilities. In Huashan, systematically following each potential set of moves should eventually lead to a solution, but it will also allow us to see if there's a pattern in the moves we need to make. In the chess board problem, one way of specialising is to ask how many 2x2 squares there are, how many 3x3 squares there are etc. If we go through this sequentially, we'll eventually be able to add up all the squares, and we might also see if there's a relationship between the size and number of squares visible.

Artful specialization is when we start choosing examples deliberately that might test a hypothesis. For example, in the handshake problem, we have a formula that works for 5 people, 6 people. Does it still hold for 1 person? How about for 0 people? This way, we can test the limits of our solution. In the painters problem, artfully specializing might be to choose some examples that will help with our intuition, for example, if both painters paint at the same rate, then if one of the painters does all the work, then somewhere in between. A carefully selected set of instances that should themselves lead to a pattern.

Sometimes by specializing we are consciously working towards generalizing. With handshakes and chessboards, we are looking for a pattern and a general rule which will allow us to solve the problem in the particular instance. In Huashan, we might wonder if a certain set of moves would work even if we have 9 holes and 4 climbers either side. We are looking for what seems to work as a rule for solving similar problems and a justification as to why it works.

However generalizing is also the act of extending the problem. We might know how to work out the handshakes problem, by adding up the sequence, but this will be hard if we

want to solve the problem for 1000 people. In the chessboard problem, we might wonder what kind of rules we could find for rectangles, or whether the rule works the same for triangular boards. Are there additional rules that might make something harder or easier? Will the trains always cross?

Generalising may be part of the review process that leads us back to different methods of attack, and it might be through this process that we gain the greatest understanding of the problem.