

[Introduction to Problem Solving]

The use of problem solving activities in mathematics classrooms has been emphasised more and more in recent decades. Mathematics is one of the most important disciplines for developing our abilities in abstract thinking and logical reasoning. The benefits of mathematical study are hence much broader than its “real-life” applications. Furthermore, knowledge of mathematical theories and formulas is of little use if one isn’t flexible and creative in adapting this understanding to the problem at hand. In fact, in many real contexts developing and refining a mathematical model or way of thinking is much more important than finding the answer.

Broadly speaking, mathematical problem solving involves the application of logical and mathematical methods to non-routine or open problems. Rather than skills exercises, such problems can usually be approached in a number of ways, but usually require more than just an “answer”. A number of formal frameworks have been identified for undertaking a problem solving task, sometimes with 9 steps, sometimes with 5, however in general they can be reduced to three phases: Entry, Attack and Review.

We’ll give an overview of these stages in this topic, however we also really want to emphasize the growth and reflection aspects that distinguish problem solving from routine exercises and skills practice – so let’s place problem solving within another framework: The Hero’s Journey.

Roughly, the hero’s journey consists of a main character, a disturbance or ‘call to adventure’, a goal, complications and potential disaster that is to be overcome. A nice thing about thinking of problem solving in this way is that: firstly, we can think of ourselves as heroic; and then, it might help us to remember some important lessons when we embark on solving a problem.

The key idea here is the call to adventure. The call to adventure is what takes the hero out of their everyday, ordinary life into the unknown. In the case of problem solving, the call to adventure comes when we are challenged by something. If it is a routine problem, then we will learn nothing, and if it does not grab our attention and curiosity, then we won't try to solve it. It's good to recognise the difference, and that when we are hooked by a problem, something special might happen.

Another important feature that we'll do our best to emphasize in this topic is that after gaining wisdom and averting the disaster (solving the problem), the hero returns with this knowledge to make the known world a better place. If we are going to embark on a journey of solving a problem, getting stuck, feeling down about our difficulties, and finally obtaining our solution, then hopefully we can use this knowledge in the future. All too often, this final aspect is overlooked and we are bound to make the same mistakes over and over.

So in problem solving, first we're confronted with the problem. In our experiences, it's most often a teacher that sets the problem and that might be our base motivation for solving it, however there are also those problems that we might discover ourselves, or that might arise in our lives or as we're thinking about other things. Often a problem will take hold when we realise that our initial thoughts were wrong – it might have seemed simple or as if it had a routine solution, and if this leads to a dead end we can become intrigued. There will then be those problems that cause a struggle. We can spend hours, days... (weeks), maybe because we don't have the required skills and understanding, maybe because we're working under some misconception about the problem. And then, after hitting a number of deadends, famous in mathematical texts that talk about problem solving is the "AHA" moment. Of course, with real problems often we'll have as many ahas that then lead to dead ends again. If we're organised, it's in this process that we make

conjectures, test them, adjust and reworking them until we can update our understanding and then trust our solution. We might have a moment of serenity (or annoyance if we feel like we should have solved it sooner and the problem got the better of us) and then if we spend the time to reflect, we should be able to think about what we learned and become better problem solvers.