

Order of Operations

*Order of operations
is part of the
grammar of
mathematics.*

The following breaks down steps for order of operations. Remember that this order is not something people should intuitively 'get', but rather a formal set of rules that have been introduced so that the evaluation of an equation is not ambiguous.

When evaluating any expression, we proceed in the following order:

Brackets - evaluate anything in brackets (using the order of operations if there are complicated expressions within the brackets) and then it's easiest to replace the bracketed part with the result when we rewrite the expression. For example,

$$(34 + 2) \times 2 + 1$$

becomes $36 \times 2 + 1$.

Powers/indices - even though powers can sometimes be thought of as repeated multiplication, we apply this operation *before* multiplying or adding. If some value is raised to a power which is an expression, the power should be evaluated first. So,

$$4 \times 3^{2+1}$$

Becomes 4×3^3

and then 4×27 .

We generally proceed from left to right – unless there is an operation that takes precedence.

Multiplication/Division - Multiplications and divisions are also performed before addition and subtraction, completed in the order they are given. So we have

$$3 + 4 \times 6 - 1 \div 4$$

becomes $3 + 24 - 0.25$.

Addition/Subtraction - Finally, we evaluate the addition and subtraction (in the order they are given).

Example

Suppose we wanted to evaluate:

$$3 - (9 - 2) \times 3^2 + 10 \div 5 \times 2$$

First we evaluate the **brackets**:

$$3 - 7 \times 3^2 + 10 \div 5 \times 2$$

Then we evaluate **powers**:

$$3 - 7 \times 9 + 10 \div 5 \times 2$$

Multiplication/division:

$$3 - 63 + 2 \times 2$$

$$3 - 63 + 4$$

Finally **addition/subtraction**:

$$-60 + 4$$

$$-56$$

*Be careful of
“implied” brackets.*

In the case of algebraic statements, we sometimes leave out some of the brackets, multiplication and division symbols. Instead of writing division symbols, division is usually indicated by writing the expressions using the vinculum (i.e. similar to fractions). Multiplication is assumed if there is a number or variable next to another variable or expression and sometimes brackets will be left out if we are performing divisions or using powers raised to an expression.

Here are some examples.

$3 \times x \times y$ would usually be written

$$3xy$$

$4 \times x \div y + 1$ would usually be written

$$\frac{4x}{y} + 1$$

$(3 + x) \div (2 + x)$ would usually be written

$$\frac{3+x}{2+x}$$

$y \times \frac{(3+a)}{(2-x)} + 4^{(x-1)}$ would usually be written

$$\frac{y(3+a)}{2-x} + 4^{x-1} .$$

These are just the conventions that are usually used. In some cases, we may decide to leave the brackets or symbols in if we are worried about the expressions being misinterpreted.

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Recommended Resource

Willers, M. *The Bedside Book of Algebra: From Vectors to Variables: the ABC of X Plus Z*. Pier 9. 2009